Code No: R1621045 **R16** SET - 1

II B. Tech I Semester Regular/Supplementary Examinations, October/November - 2018
RANDOM VARIABLES & STOCHASTIC PROCESSES
(Electronics and Communication Engineering)

Time: 3 hours Max. Marks: 70

Note: 1. Question Paper consists of two parts (**Part-A** and **Part-B)**
2. Answer **ALL** the question in **Part-A**
3. Answer any **FOUR** Questions from **Part-B**

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PART –A

1. a) Two dice are thrown. What is the probability that the sum on the dice is twelve? (2M)
   b) What is the relation between CDF and PDF of a discrete random variable? (3M)
   c) Find the mean value of a uniform random variable. (2M)
   d) Define the auto-covariance of a random process. (2M)
   e) Write $E[X^2(t)]$ in terms of the PSD of $X(t)$. (2M)
   f) When a metallic resistor R is at temperature T, random electron motion produces a noise voltage at the open circuited terminals. What is the name of this noise voltage? Give the expression for PSD noise voltage. (3M)

PART -B

2. a) Define and explain the following with an example: (7M)
      i. Equally likely events
      ii. Exhaustive events
      iii. Mutually exclusive events
   b) State and prove the properties of cumulative distribution function (CDF) of X. (7M)

3. a) Find the n^{th} moment of the random variable X in terms of its characteristic function $\Phi_X(\omega)$. (7M)
   b) Find the variance of a uniform random variable distributed over the interval [a,b]. (7M)

4. Consider random variables $Y1$ and $Y2$ related to arbitrary random variables X and Y by the coordinate rotation
$$Y1 = X\,Cos\,\theta \,+\, Y\,Sin\,\theta$$
$$Y2 \,=\, -X\,Sin\,\theta \,+\, Y Cos\,\theta$$
   a) Find the covariance of $Y1$ and $Y2$. (7M)
   b) For what value of θ, the random variables $Y1$ and $Y2$ are uncorrelated. (7M)

5. a) The auto correlation function for a stationary ergodic process with no periodic components is $R_{XX}(\tau) = 625 + \frac{16}{1+36\tau^2}$. Find mean and variance of the random process$X(t)$. (7M)
   b) Explain about Ergodicity (7M)
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6. a) Show that the auto-correlation function and power spectral density form Fourier transform pair. (7M)
   b) State all the properties of power spectral density and cross power spectral density. (7M)

7. a) Derive the relation between PSDs of input and output random processes of an LTI system. (7M)
   b) Write notes on generalized Nyquist theorem for modeling a source of thermal noise. (7M)

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PART –A

1. a) Define a random variable. (2M)

b) If 'Θ' is a Random variable, uniformly distributed on the interval (0, $\pi/2$) find the mean value of 'Θ'. (3M)

c) Give the expression for the joint PDF of bi-variate Gaussian random variable? (2M)

d) Let $Y(t) = X(t) + 2\pi$ **,** where $X(t)$ is WSS. Is $Y(t)$ WSS? (2M)

e) Define narrowband random process. (2M)

f) Give the expression for power spectral density.. (3M)

PART -B

2. a) A random experiment consists of flipping two coins. A random variable, X, is taking the number of heads. Sketch the distribution function of X. (7M)

b) Show that (7M)

$$\int_{x_1}^{x_2} f_X(x)dx = P(x_1 < X \leq x_2)$$

3. a) A random variable Y has a probability density function of the form (7M)

$$f_Y(y) = \begin{cases} Ky & 0 < y \leq 6 \\ 0 & elsewhere \end{cases}$$

i. Find the value of K for which this is a valid probability density function.
ii. Find the variance of Y .

b) Show that $Var[aX] = a^2 Var[X]$. (7M)

4. a) Two random variables X and Y have a joint probability density function given by (7M)

$$f_{XY}(x,y) = \begin{cases} Ae^{-(2x+3y)} & x \geq 0, \quad y \geq 0 \\ 0 & x < 0, \quad y < 0 \end{cases}$$

Find the correlation coefficient ρ_{XY}.

b) Derive the relation between correlation and covariance of two random variables X and Y. (7M)
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5. a) A wide-sense stationary random process has an autocorrelation function of the form $R_{XX}(\tau) = 2e^{-2|\tau|} + 4e^{-4|\tau|}$ (7M)

   i. Find the mean value of the process.
   ii. Find the mean-square value of the process.

   b) Define the mean-ergodic, autocorrelation-ergodic and ergodic random processes. (7M)

6. a) State all the properties of power spectral density function. (7M)

   b) Show that the cross-power spectral density and cross-correlation form Fourier transform pair. (7M)

7. a) The thermal agitation noise generated by a resistance can be closely approximated as white noise having a spectral density of $2kTR\ V^2/Hz$, where $k = 1.37 \times 10^{-23}$ W-s/°K is the Boltzmann constant, T is the absolute temperature in degree Kelvin, and R is the resistance in ohms. A physical resistance is paralleled by a capacitance as shown in Figure 1. (7M)

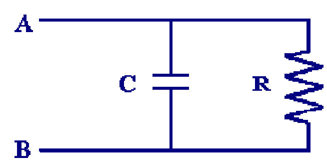

Figure 1

Calculate the mean-square value of the noise voltage across AB terminals.

   b) Write brief notes on the following: (7M)
   i. Band-limited random process
   ii. Narrowband random process

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PART –A

1. a) Define the total probability theorem. (2M)
   b) List the properties of probability mass function. (3M)
   c) Show that the first central moment is zero. (2M)
   d) Define ergodic random process. (2M)
   e) List the properties of power spectral density. (2M)
   f) Define band-limited random process. (3M)

PART -B

2. a) Define and explain the following with an example: (7M)
      i. Discrete sample space
      ii. Conditional probability
      iii. Continuous random variable
   b) A random variable, X, takes the number of heads that come-up when a fair coin is tossed twice. Obtain the probability mass function of X and plot it. (7M)

3. a) The value of a resistor, R, is uniformly distributed over the interval 990 Ω to 1010 Ω. Find the mean-square value of R. (7M)
   b) Find the probability density function of $Y = X^2$, where X is a Gaussian random variable with zero-mean and unity-variance. (7M)

4. a) The joint probability density function of two random variables X and Y is given by $f_{XY}(x, y) = \begin{cases} a(2x + y^2); & 0 \le x \le 2, 2 \le y \le 4 \\ 0; & elsewhere \end{cases}$ (7M)
   Find
      i. value of 'a'
      ii. $P(X \le 1, Y > 3)$
   b) Define central limit theorem for the following two cases: (7M)
      i. Unequal distributions
      ii. Equal distributions
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5. a) A random process is described by $X(t) = A + B\cos(\omega t + \Theta)$, where A is a random variable that is uniformly distributed between -3 and +3, B is a random variable with zero-mean and variance of 4, ω is a constant, and Θ is a random variable that is uniformly distributed from $-\frac{\pi}{2}$ to $+\frac{3\pi}{2}$. A, B and Θ are statistically independent. Calculate the variance of this process. (7M)

   b) Derive the relationship between auto-correlation function and auto-covariance function. (7M)

6. a) A stationary random process has a band-limited spectral density of the form (7M)

$$S_{XX}(f) = \begin{cases} 0.1 & |f| < 1000\ Hz \\ 0 & elsewhere \end{cases}$$

   Find the mean-square value of the random process.

   b) State the properties of cross-power spectral density functions. (7M)

7. a) Find the power spectral density of the output of the linear system with impulse response $h(t) = e^{-2t}u(t)$ if the input has a power spectral density of (7M)

$$S_{XX}(\omega) = \frac{18}{\omega^2 + 9}$$

   b) Write notes on the following: (7M)
      i. Noise equivalent bandwidth
      ii. Noise equivalent temperature

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PART –A

1. a) Define exponential and Rayleigh random variables. (2M)
   b) Define skew of a random variable. (3M)
   c) Define orthogonality. (2M)
   d) Define strict sense stationary random process. (2M)
   e) Write Wiener-Khinchin relations. (2M)
   f) Define effective Noise temperature and Noise figure (3M)

PART -B

2. a) The probability density function of a random variable is given by $f_X(x) = \frac{1}{2}\delta(x+1) + \frac{1}{2}\delta(x-1)$ (7M)
      i. Is $f_X(x)$ a valid PDF?
      ii. Find $F_X(x)$
      iii. Plot $f_X(x)$ and $F_X(x)$
   b) Write notes on the following: (7M)
      i. Rayleigh random variable ii. Uniform random variable

3. a) A random variable is described by its probability density function $f_X(x) = Ae^{-x} + \delta(x-1)$ Find the variance of X. (7M)
   b) Define characteristic function. Show that the moments of a random variable can be generated from its characteristic function. (7M)

4. a) Show that the correlation coefficient, ρ, satisfies the condition: $-1 \leq \rho \leq 1$. (7M)
   b) List the properties of jointly Gaussian random variables. (7M)

5. a) A random process is given by $X(t) = Acos(2\pi f_o t + \Theta)$ where A, f_o are real constants and Θ is a uniform random variable distributed over $(0,2\pi)$. Is $X(t)$ WSS process? (7M)
   b) List all the properties of cross-correlation functions. (7M)

6. a) Derive the Wiener-Khinchin relation for power spectra density and autocorrelation function. (7M)
   b) State and prove the properties of cross-power spectral density functions. (7M)

7. a) Show that $S_{YY}(\omega) = |H(\omega)|^2 S_{XX}(\omega)$. (7M)
   b) Explain how a source of thermal noise is modeled. (7M)

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