Code No: R1621045 **R16** SET - 1

II B. Tech I Semester Regular/Supplementary Examinations, October/November - 2019
RANDOM VARIABLES & STOCHASTIC PROCESSES
(Electronics and Communication Engineering)

Time: 3 hours Max. Marks: 70

Note: 1. Question Paper consists of two parts (**Part-A** and **Part-B)**
2. Answer **ALL** the question in **Part-A**
3. Answer any **FOUR** Questions from **Part-B**

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PART –A

1. a) Write the axioms of probability. (3M)
   b) Define Second order Moments about mean. (2M)
   c) Write the Jointly Gaussian Random density function for two random variables. (2M)
   d) Define Strict-Sense Stationarity. (2M)
   e) Write the Wiener-Khintchine relation. (2M)
   f) What is Mean value of System Response for Random Signal Response of Linear Systems? (3M)

PART -B

2. a) Gaussian random voltages X for which $a_X = 0$ and $\sigma_X = 4.2V$ appears across a 100-Ω resistor with power rating of 0.25W. What is the probability that the voltage will cause an instantaneous power that exceeds the resistor's rating? (7M)
   b) A random variable X is known to be Poisson with b=0 (7M)
      i. Plot the density and distribution functions for this random variable.
      ii. What is the probability of event $\{0 \le X \le 5\}$

3. a) A random variable X has a probability density (7M)
$$f\times(x)=\begin{cases}(1/2)\cos(x) & -\pi/2 < x < \pi/2\\ 0 & elsewhere\ in\ x.\end{cases}$$
Find the mean value of the function on $g(X)=4X^2$
   b) A random variable X is uniformly distributed on the interval $(-\pi/2, \pi/2)$. X is transformed to the new random variable Y = T (X) = *a* tan (X), where *a*> 0. Find the probability density function of Y. (7M)

4. a) For two random variables X and Y (7M)
$$f_{X,Y}(x,y) = 0.15\,\delta(x+1)\delta(y)+0.1\,\delta(x)\delta(y)+\ 0.1\,\delta(x)$$
$$\delta(y-2)+0.4\,\delta(x-1)\delta(y+2)+$$
$$0.2\,\delta(x-1)\delta(y-1)+0.5\,\delta(x-1)\delta(y-3).$$
Find the correlation coefficients of X and Y
   b) Two random variables having joint characteristic function (7M)
$\varnothing_{XY}(\omega_1, \omega_2)=\exp(-2\omega_1^2-8\omega_2^2)$. Find moment's m_{10}, mo_1, m_{11}?
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5. a) Write the properties of Autocorrelation Function of Random Process (7M)

   b) A gaussian random process is known to be a WSS process with mean $\bar{X} = 4$ and $R_{XX}(\tau)$=$25e^{-3|\tau|}$ where $\tau = \frac{|t_k - t_i|}{2}$ and i,k=1,2. Find joint Gaussian density function? (7M)

6. a) A random process had the power density spectrum (7M)

$$s_{xx}(\omega) = \frac{6\omega^2}{1+\omega^4}$$

   Find the average power in the process

   b) Assume X(t) is a wide sense stationary process with non zero mean value. show that (7M)

$$s_{xx}(\omega) = 2\pi\bar{X}^2\delta(\omega) + \int_{-\infty}^{\infty} C_{xx}(\tau)e^{-j\omega\tau}d\tau$$

   Where $C_{xx}(\tau)$ is the auto covariance function of X(t).

7. a) Define convolution. List the properties of of convolution (7M)

   b) Explain the following (7M)
      i) Noise Figure
      ii) Noise Sources

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PART –A

1. a) What are the Conditions for a Function to be a Random Variable? (2M)

b) Define Variance. (2M)

c) Write properties of Joint Density Function. (2M)

d) Define Deterministic Nondeterministic Processes with example. (3M)

e) Determine whether the below function is be valid power density spectrum? Why? (2M)

$$\frac{\cos(3\omega)}{1+\omega^2}$$

f) What is Mean-squared value of System Response? (3M)

PART -B

2. a) Define conditional probability distribution function and write the properties. (7M)

b) A random current is described by the sample space. A random variable X is defined by (7M)

$$X(i)=\begin{cases}-2 & i\le -2\\ i & -2<i\le 1\\ 1 & 1<i\le 4\\ 6 & 4<i\end{cases}$$

Show, by a sketch, the value x into which the values of i are mapped by x. What type of random variable is X?

3. a) Find mean and variance of Gaussian random variable? (7M)

b) Explain about Transformation of random variable. (7M)

4. a) Define Marginal density function? Find the Marginal density functions of below joint density function, (7M)

$$f_{XY}=\frac{1}{12}u(x)u(y)e^{-x/3}e^{-y/4}$$

b) Find the density function of W=X+Y, where the densities of X and Y are assumed to be: $f_x(x)=4u(x)e^{-4x}$; $f_y(y)=5u(y)e^{-5y.}$ (7M)
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5. a) let two random processes X(t) and Y(t) be defined by (9M)

$$X(t) = A \cos \omega_0 t + B \sin \omega_0 t$$
$$Y(t) = B \cos \omega_0 t - A \sin \omega_0 t$$

Where A and B are random variables and ω0 is a constant. Assume A and B are uncorrelated, zero mean random variables with same variance. Find the cross correlation function $R_{XY}(t,t+\tau)$.

b) Write the properties of Cross correlation Function of Random Process (5M)

6. a) Write the properties of power density spectrum (7M)

b) If X(t) is a stationary process, find the power spectrum of $Y(t) = A_0 + B_0 X(t)$ in term of the power spectrum of X(t) if A_0 and B_0 are real constants (7M)

7. a) The bandwidth of a system is 10MHz. Find the thermal noise voltage across an 800Ω resistor at room temperature. (7M)

b) If X(t) is band limited process such that $S_{xx}(\omega) = 0, when\ |\omega| > \sigma$, prove that $2[R_{xx}(0) - R_{xx}(\tau)] \leq \sigma^2 \tau^2 R_{xx}(0)$ (7M)

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PART –A

1. a) Write the applications of Gaussian random variable. (3M)
b) Define skew operation of random variable. (2M)
c) Define Marginal Distribution function and conditional Distribution function. (3M)
d) Define Mean- Ergodic Process? (2M)
e) What is Spectrum? (2M)
f) Define Thermal Noise. (2M)

PART -B

2. a) Define Random variable? Write the conditions for a function to be random variable (5M)
b) A random voltage can have any value defined by the set 'S' = {a≤ s ≤ b}. A quantizer, divides S into 6 equal-sized contiguous subsets and generates random variable X having values {-4, -2, 0, 2, 4, 6}. Each value of X is earned to the midpoint of the subset of 'S' from which it is mapped (9M)
i) Sketch the sample space and the mapping to the line that defines the values of X ii) Find a and b?

3. a) Find the expected value of the function $g(X)=X^3$ where X is a random variable defined by the density (7M)
$f_X(x)=\left(\frac{1}{2}\right)u(x)\exp(-x/2)$.
b) Let X be a Poisson random variable then Find out its mean and variance (7M)

4. a) Joint Sample Space has three elements (1, 1), (2, 2), and (3, 3) with probabilities 0.4, 0.3, 0.3 respectively then draw the Joint Distribution Function diagram. (4M)
b) Find the density function of W=X+Y, where the densities of X and Y are assumed to be: fx(x)=0.5[u(x)-u(x-2)]; fy(y)=0.25[u(y)-u(y-4)] (10M)

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5. a) Given that the autocorrelation function for a stationary Ergodic process with no period components is (7M)

$$R_{xx}(\tau) = 25 + \frac{4}{1 + 6\tau^2}$$

Find the mean and variance of process X(t)?

b) Give the random process by (7M)

$$X(t)=A \cos(w_0 t) + B \sin(w_0 t)$$

Where $w_0$is a constant, and A and B are uncorrelated zero mean random variables having different density functions but the same variance ,show that X(t) is wide sense stationary but not strictly stationary

6. a) Derive the relationship between power spectral density and autocorrelation function (7M)

b) The autocorrelation function of a random process X(t) (7M)

$$R_{xx}(\tau) = 3 + 2\exp(-4\tau^2)$$

i. Find the power spectrum of X(t)
ii. What is the average power in X(t)?

7. a) Consider a white Gaussian noise of zero mean and power spectral density ${N_o}/{2}$ applied to a low pass RC filter whose transfer function is $H(f) = \frac{1}{1+j2\pi fRC}$. find the autocorrelation function of the output random process. (7M)

b) Find the average Noise Figure of cascaded networks (7M)

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PART –A

1. a) Give Distribution function of Poisson random variable (2M)
   b) If mean of X is 3, find the mean of 3X+1? (2M)
   c) Explain the significance of central limit theorem (2M)
   d) Define Wide-Sense Stationary? (3M)
   e) What is Power density Spectrum? (2M)
   f) Write the properties of Band limited process. (3M)

PART -B

2. a) Explain the properties of Gaussian random variable. (7M)
   b) A Gaussian random variable X has $a_X = 2, and \sigma_X = 2$ (7M)
      i. Find $P\{X > 1.0\}$
      ii. Find $P\{X \leq -1.0\}$

3. a) For the binomial density function. Find the mean and variance. (7M)
   b) State and prove Chebchev's inequality? (7M)

4. a) The two random variables V and W are defined as (7M)
      V=X+aY
      W=X-aY
      Where 'a' is real number and X and Y are random variables. Determine 'a' in terms of X and Y such V and W are orthogonal?
   b) Gaussian random variables X and Y have first and second order moments (7M)
      m_{10}=-1.1, m_{20}=1.16, m_{01}=1.5, m_{02} =2.89,R_{XY}=-1.724 find C_{XY}, ρ?
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5. a) Explain the following (7M)
   i) N^{th} order stationary
   ii) Strict sense stationary
   iii) Wide sense stationary

   b) Define cross correlation function. List the properties of cross correlation function. (7M)

6. a) Derive the relationship between cross power density spectrum and cross correlation. (7M)

   b) A random process is given by $X(t) = A\cos(\Omega t + \theta)$ where A is a real constant, Ω is a random variable with density function $f_\Omega(\Omega)$ and θ is a random variable uniformly distributed over the interval $(0, 2\pi)$ independent of Ω. Show that the power spectrum of $X(t)$ is (7M)

$$S_{XX}(\omega) = \frac{\pi A^2}{2}[f_\Omega(\omega) + f_\Omega(-\omega)]$$

7. a) If the input auto correlation function is $R_{xx}(\tau) = Ae^{-\alpha|\tau|}$, where A and α are constants, find the output spectral density. (7M)

   b) The impulse response of a low pass filter is $\alpha e^{-\alpha t}U(t); where\ \alpha = \frac{1}{RC}$.if a zero mean white Gaussian process N(t) is input into this filter, find the mean square value and autocorrelation function of the output. (7M)